

Precalculus  
**Scope and Sequence**

**Quarter 1**

**Unit of Study 1.1: Parent Functions and Transformations  
(5 days)**

***Standards for Mathematical Content***

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**Building Functions**

**F-BF**

**Build new functions from existing functions** [*Include simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types*]

F-BF.3 Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

***Standards for Mathematical Practice***

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**4 Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

## 5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

## 7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

## Unit of Study 1.2: Graph Rational Functions (15 days)

### *Standards for Mathematical Content*

Interpreting Functions	F-IF
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**Analyze functions using different representations** [*Focus on using key features to guide selection of appropriate type of model function*]

- F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.\*
- d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

### *Standards for Mathematical Practice*

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**6 Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

## Unit of Study 1.3: Graphing Logarithmic and Exponential Functions (10 days)

### *Standards for Mathematical Content*

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<b>Interpreting Functions</b>	<b>F-IF</b>
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**Analyze functions using different representations** [*Focus on using key features to guide selection of appropriate type of model function*]

- F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.\*
- e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

### *Standards for Mathematical Practice*

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#### **1 Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

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## Unit of Study 1.4: Build Composite Functions (10 days)

### Standards for Mathematical Content

#### Reasoning with Equations and Inequalities

A-REI

##### Solve equations and inequalities in one variable

A-REI.4 Solve quadratic equations in one variable.

- a. Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p)^2 = q$  that has the same solutions. Derive the quadratic formula from this form.

##### Analyze functions using different representations

F-IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

- a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

### Functions

#### Building Functions

F-BF

**Build a function that models a relationship between two quantities** [*Include all types of functions studied*]

F-BF.1 Write a function that describes a relationship between two quantities.\*

- c. (+) Compose functions. *For example, if  $T(y)$  is the temperature in the atmosphere as a function of height, and  $h(t)$  is the height of a weather balloon as a function of time, then  $T(h(t))$  is the temperature at the location of the weather balloon as a function of time.*

### Standards for Mathematical Practice

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**Quarter 2****Unit of Study 2.1: Build Sequences and Series (10 days)*****Standards for Mathematical Content***

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**Building Functions****F-BF**

**Build a function that models a relationship between two quantities** [*Include all types of functions studied*]

F-BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.\*

***Standards for Mathematical Practice***

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## Unit of Study 2.2: Unit Circle and Right Triangle Trigonometry (5 days)

### *Standards for Mathematical Content*

#### Trigonometric Functions

#### F-TF

#### Extend the domain of trigonometric functions using the unit circle

- F-TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- F-TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- F-TF.3 (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for  $\pi/3$ ,  $\pi/4$  and  $\pi/6$ , and use the unit circle to express the values of sine, cosine, and tangent for  $\pi-x$ ,  $\pi+x$ , and  $2\pi-x$  in terms of their values for  $x$ , where  $x$  is any real number.
- F-TF.4 (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

### *Standards for Mathematical Practice*

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## **8 Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

## Unit of Study 2.3: Graph Trigonometric Functions (5 days)

### Standards for Mathematical Content

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Interpreting Functions	F-IF
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Analyze functions using different representations [~~Logarithmic~~ and trigonometric functions]

F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

- e. Graph ~~exponential and logarithmic functions, showing intercepts and end behavior,~~ and trigonometric functions, showing period, midline, and amplitude.

### Standards for Mathematical Practice

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#### 2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

#### **4 Model with mathematics.**

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## Unit of Study 2.4: Inverse Trigonometric Functions (10 days)

### *Standards for Mathematical Content*

<b>Building Functions</b>	<b>F-BF</b>
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#### **Build new functions from existing functions**

F-BF.4 Find inverse functions.

- b. (+) Verify by composition that one function is the inverse of another.
- c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
- d. (+) Produce an invertible function from a non-invertible function by restricting the domain.

<b>Trigonometric Functions</b>	<b>F-TF</b>
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#### **Model periodic phenomena with trigonometric functions**

F-TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.\*

F-TF.6 (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

F-TF.7 (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.\*

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## Unit of Study 2.5: Prove Trigonometric Identities (10 days)

### Standards for Mathematical Content

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<b>Trigonometric Functions</b>	<b>F-TF</b>
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#### Prove and apply trigonometric identities

F-TF.8 Prove the Pythagorean identity  $\sin^2(\theta) + \cos^2(\theta) = 1$  and use it to find  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  given  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  and the quadrant of the angle.

F-TF.9 (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

### Standards for Mathematical Practice

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### **3 Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

## Quarter 3

### Unit of Study 3.1: Use Conic Sections—Ellipses and Hyperbolas (10 days)

#### *Standards for Mathematical Content*

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<b>Expressing Geometric Properties with Equations</b>	<b>G-GPE</b>
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**Translate between the geometric description and the equation for a conic section**

G-GPE.3 (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

#### *Standards for Mathematical Practice*

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#### **4 Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

#### **7 Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

**8 Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

## Unit of Study 3.2: Cavalieri's Principle (5 days)

### Standards for Mathematical Content

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<b>Geometric Measurement and Dimension</b>	<b>G-GMD</b>
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#### Explain volume formulas and use them to solve problems

G-GMD.2 (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

### Standards for Mathematical Practice

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#### 1 Make sense of problems and persevere in solving them.

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#### 2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

## Unit of Study 3.3: Perform Operations with Complex Numbers (15 days)

### Standards for Mathematical Content

<b>The Complex Number System</b>	<b>N-CN</b>
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**Perform arithmetic operations with complex numbers.**

N-CN.3 (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

**Represent complex numbers and their operations on the complex plane.**

N-CN.4 (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

N-CN.5 (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,  $(-1 + \sqrt{3}i)^3 = 8$  because  $(-1 + \sqrt{3}i)$  has modulus 2 and argument  $120^\circ$ .*

N-CN.6 (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

**Use complex numbers in polynomial identities and equations.**

N-CN.8 (+) Extend polynomial identities to the complex numbers. *For example, rewrite  $x^2 + 4$  as  $(x + 2i)(x - 2i)$ .*

### Standards for Mathematical Practice

### 3 Construct viable arguments and critique the reasoning of others.

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## 7 Look for and make use of structure.

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## Unit of Study 3.4: Use Properties to Represent Complex Numbers Geometrically (10 days)

### Standards for Mathematical Content

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The Complex Number System	N-CN
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N-CN.5 (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,  $(-1 + \sqrt{3}i)^3 = 8$  because  $(-1 + \sqrt{3}i)$  has modulus 2 and argument  $120^\circ$ .*

### Standards for Mathematical Practice

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#### 6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.



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**Quarter 4****Unit of Study 4.1: Solve Systems of Equations with Matrices and Inverse Matrices (5 days)*****Standards for Mathematical Content***

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**Reasoning with Equations and Inequalities****A-REI****Solve systems of equations**

A-REI.8 (+) Represent a system of linear equations as a single matrix equation in a vector variable.

A-REI.9 (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension  $3 \times 3$  or greater).

***Standards for Mathematical Practice***

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## 5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

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## Unit of Study 4.2: Perform Operations with Matrices (5 days)

### *Standards for Mathematical Content*

<b>Vector and Matrix Quantities</b>	<b>N-VM</b>
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#### **Perform operations on matrices and use matrices in applications.**

N-VM.8 (+) Add, subtract, and multiply matrices of appropriate dimensions.

N-VM.9 (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

N-VM.10 (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

N-VM.12 (+) Work with  $2 \times 2$  matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

### *Standards for Mathematical Practice*

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## **4 Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

## Unit of Study 4.3: Use Matrices in Real-World Applications (10 days)

### *Standards for Mathematical Content*

<b>Vector and Matrix Quantities</b>	<b>N-VM</b>
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#### **Perform operations on matrices and use matrices in applications.**

N-VM.6 (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

N-VM.7 (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.

### *Standards for Mathematical Practice*

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## Unit of Study 4.4: Represent and Model with Vector Quantities (5 days)

### *Standards for Mathematical Content*

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<b>Vector and Matrix Quantities</b>	<b>N-VM</b>
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**Represent and model with vector quantities.**

- N-VM.1 (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g.,  $\mathbf{v}$ ,  $|\mathbf{v}|$ ,  $\|\mathbf{v}\|$ ,  $v$ ).
- N-VM.2 (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
- N-VM.3 (+) Solve problems involving velocity and other quantities that can be represented by vectors.

**Perform operations on matrices and use matrices in applications.**

- N-VM.11 (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

### *Standards for Mathematical Practice*

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## Unit of Study 4.5: Perform Operations with Vectors (10 days)

### Standards for Mathematical Content

Vector and Matrix Quantities	N-VM
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#### Perform operations on vectors.

N-VM.4 (+) Add and subtract vectors.

- Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
- Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
- Understand vector subtraction  $\mathbf{v} - \mathbf{w}$  as  $\mathbf{v} + (-\mathbf{w})$ , where  $-\mathbf{w}$  is the additive inverse of  $\mathbf{w}$ , with the same magnitude as  $\mathbf{w}$  and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

### Standards for Mathematical Practice

#### 2 Reason abstractly and quantitatively.

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## 7 Look for and make use of structure.

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## Unit of Study 4.6: Multiply a Vector by a Scalar (5 days)

### Standards for Mathematical Content

#### Vector and Matrix Quantities

N-VM

#### Perform operations on vectors.

N-VM.5 (+) Multiply a vector by a scalar.

- Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as  $c(v_x, v_y) = (cv_x, cv_y)$ .
- Compute the magnitude of a scalar multiple  $c\mathbf{v}$  using  $\|c\mathbf{v}\| = |c|\mathbf{v}$ . Compute the direction of  $c\mathbf{v}$  knowing that when  $|c|\mathbf{v} \neq 0$ , the direction of  $c\mathbf{v}$  is either along  $\mathbf{v}$  (for  $c > 0$ ) or against  $\mathbf{v}$  (for  $c < 0$ ).

### Standards for Mathematical Practice

#### 1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

#### 4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

**8 Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

