Algebra 1, Quarter 1, Unit 1.4

Solving Linear Equations and Inequalities

Overview

Number of instructional days: 6 (1 day = 45–60 minutes)

Content to be learned

- Justify each step in solving a simple equation.
- Prove solution methods using a viable argument.
- Solve linear equations and inequalities in one variable, including literal equations.
- Understand that the graph of a linear equation represents the relationship of two variables.
- Use word problems to create equations and inequalities.
- Create and graph on a coordinate plane equations with independent and dependent variables.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Explain the steps they took in order to solve a linear equation or inequality.
- Prove that the steps they took to solve a problem are viable through various methods.

Model with mathematics.

- Solve problems arising in everyday life, society, and the workplace by creating equations and inequalities.
- Create and interpret information on a graph of equations and inequalities.
- Use appropriate tools strategically.
- Use pencil and paper, concrete models, a ruler, a calculator, or a computer algebra system to graph equations on coordinate axes with labels and scales.

Essential questions

- What are the similarities and differences between solving a problem algebraically and solving it graphically?
- How does rearranging formulas help you solve real-world problems?
- How does justifying your steps help to determine that your solution to the equation is correct?
- What are the similarities and differences in solving one-variable and two-variable equations?

Written Curriculum

Common Core State Standards for Mathematical Content

Reasoning with Equations and Inequalities	**A-REI**

Understand solving equations as a process of reasoning and explain the reasoning

A-REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Solve equations and inequalities in one variable

A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Represent and solve equations and inequalities graphically [*Linear and ~~exponential~~; learn as general principle*]

A-REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Creating Equations★	**A-CED**

Create equations that describe numbers or relationships [*Equations using all available types of expressions, including simple root functions*]

A-CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear ~~and quadratic~~ functions~~, and simple rational and exponentia~~l functions.*★

A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

A-CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*★

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Clarifying the Standards

Prior Learning

In kindergarten, students explained thinking by making a drawing. In first grade, students began to solve simple word problems. In third grade, students solved two-step word problems using letters for unknown quantities. In fourth grade, students began solving multistep word problems. In fifth grade, students graphed the ordered pairs on a coordinate plane. In grade 6, students learned that the solutions to an equation are the values of the variables that make the equation true. Students used properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. They also constructed and analyzed tables, such as tables of quantities that were in equivalent ratios, and they used equations (such as $3x = y$) to describe relationships between quantities. In grade 7, students used the arithmetic of rational numbers as they formulated expressions and equations in one variable and used these equations to solve problems. Seventh-grade students graphed proportional relationships and understood the unit rate informally as a measure of the steepness of the related line, called the slope. In grade 8, students used linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognized equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understood that the slope (m) of a line is a constant rate of change, so that, if the input, or x-coordinate, changes by an amount A, the output, or y-coordinate, changes by the amount $(m)A$. Interpreting the model in the context of the data required students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation. Eighth-grade students also strategically chose and efficiently implemented procedures to solve linear equations in one variable, understanding that when they used the properties of equality and the concept of logical equivalence, they maintained the solutions of the original equation. Students used linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

Current Learning

The content and concepts in this unit are a critical area for algebra 1 students. By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. This unit builds on these earlier experiences by asking students to analyze and explain the process of solving an equation. Students develop fluency in writing, interpreting, and translating among various forms of linear equations and using them to solve problems. They master the solution of linear equations and apply related solution techniques. All of this work is grounded in understanding quantities and relationships between them. Students should focus on and master standard A.REI.1 for linear equations and be able to extend and apply their reasoning to other types of equations in future courses.

Future Learning

In geometry, students will use equations to help solve problems with congruence and proportions. Students will solve equations and the process of reasoning will be expanded in algebra 2 for extraneous solutions. Students in algebra 2 will expand their ability to create equations that describe numbers or relationships to include root functions. They will also build new functions from existing functions, including simple radical, rational, and exponential functions, with an emphasis on the common effect of each transformation across function types. Students will solve exponential equations with logarithms in algebra 2. Solving systems of equations is used in all subsequent mathematical courses. Engineers in every field, computer programmers and software engineers, biological scientists, economists, urban and regional planners, and optometrists and pharmacists use these methods in their careers.

Additional Findings

In *A Research Companion to Principles and Standards for School Mathematics*, Chazan and Yerushalmy discuss the cognitive difficulties that many students have in working with the complex relationships embedded in systems of equations. As an example, they describe the methods that students must use to solve a system of equations consisting of a linear equation in standard form and a circle in standard form. As students work through solving such a system, they must move “from an equation in two variables to a function of one” to enable use of the substitution algorithm “from an equation in two variables to an equation in one variable” *using* the algorithm, to generating equivalent expressions in solving the new equation. Chazan and Yerushalmy indicate that this complexity is common in learning about equivalence in school algebra, and that this cognitive complexity must be taken into account when approaching topics involving equivalence (pp.129-131). Chazan and Yerushalmy also indicate that graphing technology can assist students in making sense of equivalent expressions. (p. 130)